$$\frac{d}{dx}\left(x^n\right) = nx^{n-1}$$

the derivative of x^2:

$$\frac{(x+h)^2 - x^2}{h} = \frac{\left(x^2 + 2xh + h^2\right) - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h \xrightarrow[\text{reeeally small}]{\text{as h gets}} 2x$$

the derivative of x^3:

$$\frac{(x+h)^3 - x^3}{h} = \frac{\left(x^3 + 3x^2h + 3xh^2 + h^3\right) - x^3}{h} = \frac{3x^2h + 3xh^2 + h^3}{h} = 3x^2 + 3xh + h^2 \xrightarrow[\text{reeeally small}]{\text{as h gets}} 3x^2$$

the derivative of x^4:

$$\frac{(x+h)^4 - x^4}{h} = \frac{\left(x^4 + 4x^3h + 6x^2h^2 + 4xh^3 + h^4\right) - x^4}{h} = \frac{4x^3h + 6x^2h^2 + 4xh^3 + h^4}{h} = 4x^3 + 6x^2h + 4xh^2 + h^3 \xrightarrow[\text{reeeally small}]{\text{as h gets}} 4x^3$$

the derivative of x^5:

$$\frac{(x+h)^5 - x^5}{h} = \frac{\left(x^5 + 5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5\right) - x^5}{h} = \frac{5x^4h + 10x^3h^2 + 10x^2h^3 + 5xh^4 + h^5}{h} = 5x^4 + 10x^3h + 10x^2h^2 + 5xh^3 + h^4 \xrightarrow[\text{reeeally small}]{\text{as h gets}} 5x^4$$

$\vdots$ $\quad$ $\vdots$ $\quad$ $\vdots$ $\quad$ $\vdots$ $\quad$ $\vdots$ $\quad$ $\vdots$

the derivative of x^n:

$$\frac{(x+h)^n - x^n}{h} = \frac{\left(x^n + nx^{n-1}h + \left(\text{a bunch of cross terms with } x\text{'s and } h\text{'s}\right)\right) - x^n}{h} = \frac{nx^{n-1}h + \left(\text{a bunch of cross terms with } x\text{'s and } h\text{'s}\right)}{h} = nx^{n-1} + \left(\begin{array}{c}\text{a bunch of cross terms with } x\text{'s and } h\text{'s, but with the } h \text{ power down by 1}\end{array}\right) \xrightarrow[\text{reeeally small}]{\text{as h gets}} nx^{n-1}$$

$$= \frac{\left(x^n + nx^{n-1}h + \binom{n}{n-2}x^{n-2}h^2 + \binom{n}{n-3}x^{n-3}h^3 + \cdots + \binom{n}{2} + nxh^{n-1} + h^n\right) - x^n}{h} = \frac{nx^{n-1}h + \binom{n}{n-2}x^{n-2}h^2 + \binom{n}{n-3}x^{n-3}h^3 + \cdots + \binom{n}{2} + nxh^{n-1} + h^n}{h} = nx^{n-1} + \binom{n}{n-2}x^{n-2}h + \binom{n}{n-3}x^{n-3}h^2 + \cdots + \binom{n}{2} + nxh^{n-2} + h^{n-1} \xrightarrow[\text{reeeally small}]{\text{as h gets}} nx^{n-1}$$

$$= \frac{\overbrace{\left(\sum_{k=0}^{k=n}\binom{n}{n-k}x^{n-k}h^k\right)}^{\text{the binomial theorem!!!}} - x^n}{h} = \frac{\left(x^n + \binom{n}{n-1}x^{n-1}h^1 + \sum_{k=2}^{k=n}\binom{n}{n-k}x^{n-k}h^k\right) - x^n}{h} = \frac{nx^{n-1}h + \sum_{k=2}^{k=n}\binom{n}{n-k}x^{n-k}h^k}{h} = nx^{n-1} + \sum_{k=2}^{k=n}\binom{n}{n-k}x^{n-k}h^{k-1} \xrightarrow[\text{reeeally small}]{\text{as h gets}} nx^{n-1}$$